**Requirements**

1. Formally **define the predicate** that will be proved inductively.
2. Prove that the predicate holds in the **base case**.
3. Formally state the **inductive hypothesis**.
4. Assume the inductive hypothesis, and prove the **inductive step**.
5. **Conclude** that the predicate holds in general.

**Example**

Prove that for all integers $n \geq 1$,

$$1 + 2 + \cdots + n = \frac{n(n + 1)}{2}.$$

**Proof**

**Predicate.** Let $P(n)$ be the predicate “$1 + 2 + \cdots + n = \frac{n(n + 1)}{2}$.”

**Base case.** Because $1 = \frac{1(2)}{2}$, we see that the base case $P(1)$ holds.

**Inductive Hypothesis.** Assume the inductive hypothesis, that for a particular $k$, $P(k)$ is true. That is, assume

$$1 + 2 + \cdots + k = \frac{k(k + 1)}{2}.$$  

**Inductive Step.** We must prove $P(k + 1)$. That is, we must prove that

$$1 + 2 + \cdots + k + (k + 1) = \frac{(k + 1)((k + 1) + 1)}{2}.$$  

Invoking the inductive hypothesis, we can compute the sum as follows.

$$1 + 2 + \cdots + k + (k + 1) = (1 + 2 + \cdots + k) + (k + 1) = \frac{k(k + 1)}{2} + (k + 1) = \frac{k(k + 1) + 2(k + 1)}{2} = \frac{(k + 2)(k + 1)}{2} = \frac{(k + 1)((k + 1) + 1)}{2}.$$  

This is exactly what we needed to show.

**Conclusion.** Because $P(1)$ is true, and because, for all $k$, $P(k)$ implies $P(k + 1)$, we conclude that $P(n)$ is true for all integers $n \geq 1$.  

\[\square\]