Proving Equivalence Relation

Requirements

- Prove that the relation is reflexive, symmetric, and transitive.
- Conclude that the relation is an equivalence relation.

Example

Prove that the following relation is an equivalence relation:
For all $x, y \in \mathbb{R}, xRy \Leftrightarrow |x| = |y|$.

Proof. We show that R is reflexive, symmetric, and transitive.

- Reflexive: Choose any $m \in \mathbb{R}$. mRm because $|m| = |m|$. Thus, R is reflexive.
- Symmetric: Choose any $m, n \in \mathbb{R}$. Let mRn. This implies $|m| = |n|$. We must show $mRn \Rightarrow nRm$. nRm implies $|n| = |m|$ which is true because of the symmetric property of equality. Thus, R is symmetric.
- Transitive: Choose any $a, b, c \in \mathbb{R}$. Let aRb and bRc. This implies $|a| = |b|$ and $|b| = |c|$. By the transitive property of equality, this implies that $|a| = |c|$ so aRc. Thus, R is transitive.

Because R is reflexive, symmetric, and transitive, R is an equivalence relation. □