

Requirements

1. Statement of the contrapositive
2. Proof of the contrapositive
3. Conclusion that proof of the contrapositive proves the original statement

Example

If a sum of two real numbers is less than 50, then at least one of the numbers is less than 25.

Proof. The statement will be proven by contraposition. Contrapositive of original statement: If there are two real numbers, neither of which is less than 25, then their sum will be greater than or equal to 50.

The original statement, in symbols

$$\forall x, y \in \mathbb{R} \text{ s.t. } (x + y < 50) \Rightarrow ((x < 25) \vee (y < 25))$$

The contrapositive, in symbols

$$\forall x, y, \neg((x < 25) \vee (y < 25)) \Rightarrow \neg(x + y < 50)$$

Assume for some real numbers a and b , $a \geq 25$ and $b \geq 25$. Then

$$\begin{aligned} a + b &\geq 25 + 25 \\ &\geq 50 \end{aligned}$$

This proves that two numbers, neither of which is less than 25, will sum to a value greater than or equal to 50. By proving the contrapositive, a logically equivalent statement, we have also proved that if the sum of two real number is less than 50, then at least one of the numbers is less than 25. \square