Requirements

1. State the assumption that will lead to a contradiction.
2. Show that the assumption leads to a contradiction.
3. Conclude that the assumption is false.

Example

Prove that there are infinitely many primes.

Proof. Assume, for the sake of contradiction, that there are finitely many primes. Let k be the number of primes, and let the primes be $p_1, p_2, \ldots, p_k$. Note that 1 is not prime, so each prime is greater than 1. Consider the number

$$N = p_1 p_2 \ldots p_k + 1$$

N is either prime or composite.

Case 1: N is prime.

For each p_i, $N > p_i$. Thus, there so no p_i such that $N = p_i$. So N is not one of the k primes, so there are at least $k+1$ primes. This is a contradiction, because we assumed there were only k primes.

Case 2: N is composite.

N must have some prime factor. But note that for each p_i, $N = mp_i + 1$ for some integer m. Since $p_i > 1$, $p_i \nmid N$. So whatever prime factor N has, it cannot be one of the k primes on the list. This is a contradiction, because we assumed there were only k primes.

Each case leads to a contradiction, so our original assumption that there were finitely many primes was wrong, and there must be infinitely many primes. □