Logical Equivalences

Given propositions p, q, and r, a tautology $\mathbf{t}$, and a contradiction $\mathbf{c}$, the following logical equivalences hold.

Commutative Laws:

$$p \wedge q \equiv q \wedge p$$
$$p \vee q \equiv q \vee p$$

Associative Laws:

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$
$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$

Distributive Laws:

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$
$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

Itentity Laws:

$$p \wedge \mathbf{t} \equiv p$$
$$p \vee \mathbf{c} \equiv p$$

Negation Laws:

$$p \vee \neg p \equiv \mathbf{t}$$
$$p \wedge \neg p \equiv \mathbf{c}$$

Double Nagation Law:

$$\neg(\neg p) \equiv p$$

Idempotent Laws:

$$p \wedge p \equiv p$$
$$p \vee p \equiv p$$

Universal Bound Laws:

$$p \vee \mathbf{t} \equiv \mathbf{t}$$
$$p \wedge \mathbf{c} \equiv \mathbf{c}$$

De Morgan's Laws:

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$
$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Absorption Laws:

$$p \vee (p \wedge q) \equiv p$$
$$p \wedge (p \vee q) \equiv p$$

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Negation of t and c:

$$\neg \mathbf{t} \equiv \mathbf{c}$$
$$\neg \mathbf{c} \equiv \mathbf{t}$$

Definition of Conditional:

$$p \to q \equiv \neg p \lor q$$

Definition of Biconditional:

$$\begin{aligned} p \leftrightarrow q &\equiv (\neg p \lor q) \land (\neg q \lor p) \\ &\equiv (p \land q) \lor (\neg p \land \neg q) \end{aligned}$$